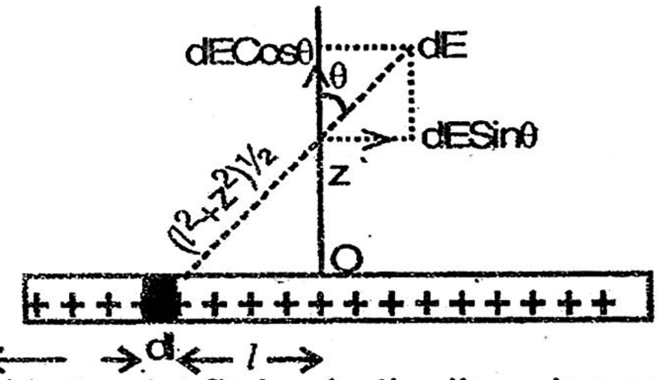
Q.1. Establish an expression for the electric field of a linear charged conducting straight wire? (2007, 2012)(SU-2016)

Sol. Let us consider a rod of length L with charge q uniformaly distributed over its entire length. The charge density on the rod is λ = q/L. We shall calculate the electric field at a point P at a distance Z from the rod. Consider a line element dl at a *distance* l from the point O. The charge on this element is dq=λ dl.

माना L लम्बाई की एक छड़ जिस की लम्बाई के अनुदिश एक समान q आवेश वितरित है। तब छड़ पर आवेश घनत्व q/L होगा। अब हम छड़ से z दूरी पर स्थित किसी बिन्दू P पर विद्युत क्षेत्रा की गणना करेंगे। माना एक रेखीय अल्पांश dl छड़ के बिन्दू O से t दूरी पर है। तब इस रेखीय अल्पांश पर आवेश है dq=ldl होगा ।



The electric field at point P due to the line element is

अतः इस रेखीय अल्पांश के कारण बिन्दु P पर विद्युत क्षेत्र है

$$dE = \frac{1}{4\pi\varepsilon_0}\left(\frac{dq}{l^2+z^2}\right)$$

$$dE = \frac{\lambda q}{4\pi\varepsilon_0}\frac{dl}{\left(l^2+z^2\right)} \text{----------------(1)}$$

The components of dE parallel and perpendicular to the rod are

$$dE_{II} = dE\,Sin\theta$$
$$dE_{\perp} = dE\,Cos\theta$$

Since an element of charge is symmetrically situated on the other side of point O. Hence the total electric field parallel to the rod is zero then one gets.

चूँकि एक आवेशीय अल्पांश बिन्दु O के दूसरी तरफ भी स्थित है। अतः छड़ के समांतर कुल विद्युत क्षेत्र शून्य होगा। तब हम पाते है

$$\int dE_{II} = \int dE\,Sin\theta = 0$$

Now the electric field of entire rod is given by

$$E = \int dE_{\perp} = dE\,Cos\theta$$

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{\left(l^2 + z^2\right)} Cos\theta$$

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{dl}{\left(l^2 + z^2\right)} \frac{Z}{\left(l^2 + z^2\right)^{1/2}}$$

If the rod is of infinite length then.

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty} \frac{z\,dl}{\left(l^2 + z^2\right)^{3/2}}$$

Or $$E = 2\frac{\lambda}{4\pi\in_0}\int_0^{\infty} \frac{z\,dl}{(l^2 + z^2)^{\frac{3}{2}}}$$

Now from figure

$l = z \tan\theta$

$\Rightarrow dl = z \sec^2\theta d\theta$

Making the use of this substitution one gets

$$E = \frac{\lambda z}{2\pi\in_0}\int_0^{\frac{\pi}{2}} \frac{z\sec^2\theta d\theta}{(z^2 + z^2\tan^2\theta)^{\frac{3}{2}}}$$

or $$E = \frac{\lambda z}{2\pi\in_0}\int_0^{\frac{\pi}{2}} \frac{z\sec^2\theta d\theta}{z^3(1 + \tan^2\theta)^{\frac{3}{2}}}$$

or $$E = \frac{\lambda}{2\pi\in_0 z}\int_0^{\frac{\pi}{2}} \frac{\sec^2\theta}{\sec^3\theta} d\theta$$

or $$E = \frac{\lambda}{2\pi\in_0 z}\int_0^{\frac{\pi}{2}} Cos\theta d\theta$$

or $$E = \frac{\lambda}{2\pi\in_0}\frac{1}{z}$$

Thus electric field a point charged rod varies inversely as distance from the rod. अतः किसी बिन्दु आवेशित छड़ का विद्युत क्षेत्र छड़ से उसकी दूरी के व्युत्क्रमानुपाती होता है।

Q. 2. Write short notes on electrostatic energy.

Sol. The electric Potential energy of a system of point charges is measured by the amount of work in bringing the charges from infinity.

बिन्दु आवेशों के किसी निकाय की वैद्युत स्थितिज उर्जा उस कार्य के बराबर होती है जो उन आवेशों को अनन्त से परस्पर समीप लाकर निकाय की रचना करने में किया जाता है।

Let us consider a system of two charges q_1 and q_2 separated by a distance r_{12} Let us imagine that the charge q_2 has been removed to infinity. The potential at the point B due to charge q_1 is

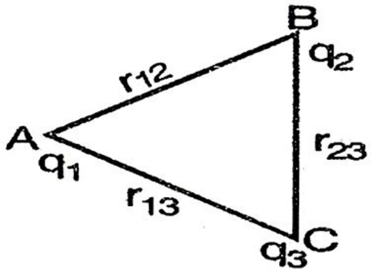
माना दो आवेशों q_1 तथा q_2, जो कि एक दूसरे से r_{12} दूरी पर है, का एक निकाय है माना आवेश q_2 को अनन्त पर प्रतिस्थापित किया जाता है। बिन्दु B पर आवेश q_1 द्वारा उत्पन्न विभव होगा

$$V = \frac{1}{4\pi \in_0} \frac{q_1}{r_{12}} \qquad \text{..............} \quad (1)$$

The work required to bring q_2 from its original point is

$$W = q_2 V$$

$$\text{or } W = \frac{1}{4\pi E_0} \frac{q_1 \, q_2}{r_{12}} \qquad \text{.................}(2)$$



By definition it is the electric potential energy U of the system (q_1 & q_2). Thus

परिभाषा से यह निकाय (q_1+q_2)की वैद्युत स्थितिज ऊर्जा U होगी। अतः

$$U = \frac{1}{4\pi \in_0} \frac{q_1 \, q_2}{r_{12}} \qquad \text{..........................}(3)$$

If another charge q_3 is brought at a distance r_{13} from q_1 and r_{23} from q_2, keeping q_1 and q_2 fixed, then the work done is given by

यदि कोई अन्य आवेश q_3 को q_1 से r_{13} दूरी पर तथा q_2 से r_{23}, q_1,q_2 को स्थिर रखते हुए लाया जाय तो किया गया कार्य होगा ......

$$\frac{1}{4\pi \in_0}\left(\frac{q_1 \, q_3}{r_{13}} + \frac{q_2 \, q_3}{r_{23}}\right)$$

Hence the total potential energy of the system is

$$U_3 = \frac{1}{4\pi \in_0}\left(\frac{q_1 \, q_2}{r_{12}} + \frac{q_1 \, q_3}{r_{13}} + \frac{q_2 \, q_3}{r_{23}}\right)$$

In general, if we have n different charges in space then the potential energy of the system is

अतः व्यापक रुप से यदि हम विभिन्न आवेशो को ले तो निकाय की स्थितिज ऊर्जा होगी।

$$U = \frac{1}{2} \frac{1}{4\pi \in_0} \sum_{i=1}^{n} \sum_{j=1} \frac{q_i \, q_j}{r_{ij}}$$

Q. 3. State and prove the Gauss's Law of Electrostatics. Using this law calcalate the electric field at the external and internal point of a charged sphere?

([illegible]) OR

State the integral form of Gauss law in electrostatics in term of charge density.

([illegible] 2011, 2013, 2014, 2015, 2016)

Sol. Gauss's Law : According to this law the total electric flux through a closed surface is equal to [illegible] times the net charge enclosed within the surface i.e

गॉस के नियम के अनुसार ^ किसी बन्द पृष्ठ से गुजरने वाली कुल विद्युत फ्लक्स बन्द पृष्ठ के कुल आवेश का $1/e_0$ गुना होता है।'

$$\oint E.ds = \frac{1}{\epsilon_0}\sum qi = \frac{q}{\epsilon_0}$$

where Σ qi is the total charge enclosed within the surface

PROOF : Consider a closed surface S enclosing a charge distribution Let us calculate the flux of electric field due to a charge element dq =ρ dv where ρ is the volume density of charge. Consider an element of surface ds. The electric flux due to charge dq through this elementary surface ds is E_1.ds. where E_1 is the electric field due to dq at the element of surface ds

Thus the electric flux due to charge q_1 through the entire surface is

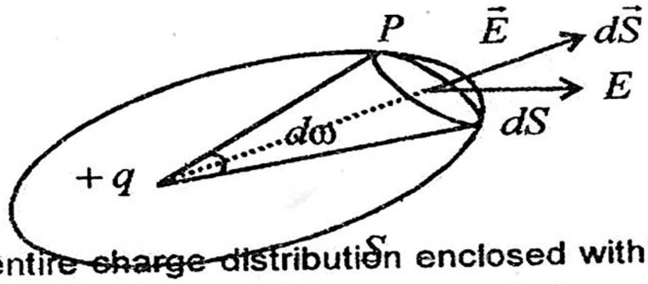
माना एक बन्द पृष्ठ s है जिसमे आवेश का वितरण है। अब हम एक आवेश तत्व dq= r dv के द्वारा विद्युत क्षेत्रा के फ्लक्स की गणना करेंगे। जहा q आवेश का आयतन घनत्व है। माना एक छोटा आवेश तत्व जिसका पृष्ठ ds है। आवेश dq के द्वारा इस पृष्ठ पर विद्युत फ्लक्स E_1 ds, होगा। जहाँ E_1 आवेश dq द्वारा इस पृष्ठ का विद्युत क्षेत्र है।

अतः किसी आवेश q के द्वारा सम्पूर्ण पृष्ठ पर विद्युत फ्लक्स होगा–

$$d\phi = \oint_s E.\,ds = \oint_s \frac{1}{4\pi\,\epsilon_0}\frac{dq}{r^2}\,.ds \text{ or } d\phi = \frac{1}{4\pi E_0}\frac{dq}{r^2}\oint ds$$

$$\text{or } d\phi = \frac{1}{4\pi E_0}\frac{dq}{r^2}4\pi r^2$$

$$\text{or } d\phi = \oint E.\,ds = \frac{dq}{\epsilon_0} \;.......(1)$$



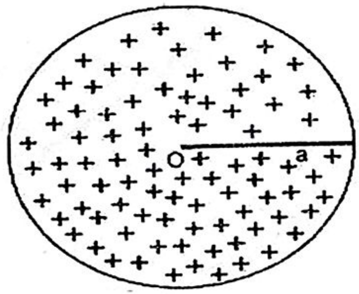
Thus the total electric flux due to the entire charge distribution enclosed within the surface is

$$\phi = \oint E.\,ds = \frac{\sum q}{\epsilon_0}$$

It is the Gauss's law of electrostatics.

Suppose we have a sphere of radius 'a' with total charge q uniformly distributed over its volume. The volume density of charge distribution is

माना एक त्रिज्या का एक गोला है जिसके आयतन के तहत कुल आवेश q एक समान वितरित है। आवेश वितरण का आयतन घनत्व होगा।



Fig

$$\rho = \frac{q}{\frac{4}{3}\pi a^3}$$

we have to calculate the electric field of this sphere at internal and external points.

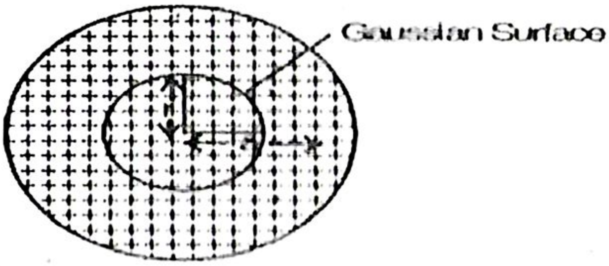
अब हम इस गोले के आन्तरिक तथा बाह्रय बिन्दुओं पर विद्युत क्षेत्र की गणना करेंगे।

***(i) Internal Point :** Let us calculate the electric field of a charged sphere at a distance r (r<a) from the centre, Construct a spherical gaussian surface of radius r.*

Total charge inside the Gaussian surface is $\frac{4}{3}\pi r^3 \rho$. Now applying the Gauss's law then, one gets

माना एक आवेशित गोले के केन्द्र से r दूरी पर (r<a)विद्युत क्षेत्रा की गणना करना है। एक r त्रिाज्य का गोलाकार गॉसियन पृष्ठ निर्मित किजिए। गॉसियन पृष्ठ के अन्दर कुल आवेश होगा 4/3 r^3 r होगा।

अब गॉस के नियम का प्रयोग करने पर हम पाते है कि



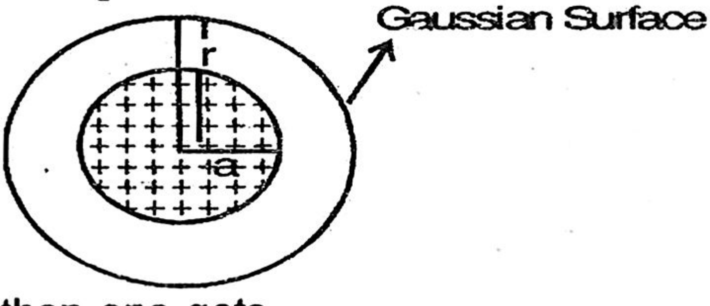
$$\oint E.ds = \frac{1}{\epsilon_0}\left(\frac{4}{3}\pi\ r^3\rho\right)$$

$$\text{or } E.4\pi r^2 = \frac{1}{\varepsilon_0}\frac{4}{3}\pi r^3\rho$$

$$\Rightarrow \quad E = \frac{\rho r}{3\varepsilon_0}$$

(ii) External Points :If the point lies outside the sphere, then one construct the Gaussian surface passing through that point. In this case, Gaussian surface encloses the entire charge (q) of the sphere.

यदि बिन्दु गोले के बाहर स्थित हो तब गॉसियन पृष्ठ उस बिन्दु से गुजरते हुए निर्मित होगा। इस केस में गॉसियन पृष्ठ गोले के समस्त आवेश q को धरण करेगा।



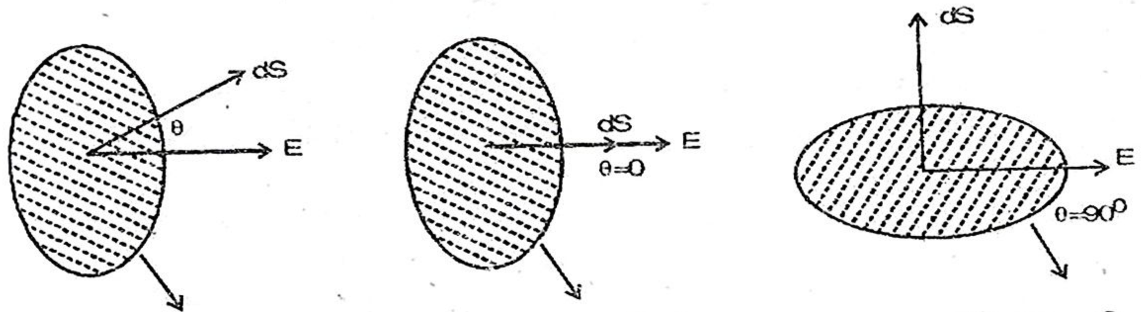
Applying Gauss's law then one gets

$$\oint E.ds = \frac{q}{\epsilon_0}$$

$$\Rightarrow E\ 4\pi r^2 = \frac{q}{\epsilon_0} \qquad \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2}}$$

Q.4. What does an eletric flux represent ? Establish an expression on the flux lined with a plane lamina of area $\vec{A}$ placed in an eletric field $\vec{E}$. What is the flux linked when $\vec{A}$ is parallel to $\vec{E}$. (2005,2011)

Sol: Let us consider an electric field in a certain regaion . At any point let electric field is E .Now consider an element of area ds at that point. The scalar product of E and ds i.e.E.ds (=E ds cosθ) is difined as the flux of electric field .over sarface ds where θ is the angle between E and normal to the area ds. Thus the electric flux is equal to the product of the normal componant of electric field and the magnitude of the area.



Flux=E.ds=E ds cosθ Flux=E.ds Flux = 0

If we have a large suface S and the electric field various from point to point over the surface S, then the electric flux over the surface is given by the surface integral.

$$\text{Flux} = \phi = \int_S E\,ds = \int_S E\cos\theta\,ds \qquad \text{............(1)}$$

If the given surface S is closed one , the electric flux is written as.

$$\text{flux } \phi = \oint_S E\,ds = \oint_S E\cos\theta\,ds \qquad (2)$$

OQ. 5. What does an electrostaic pressure represent ?
(GKP-2005,07,09,2014,2016)

Sol. A charged surface because of its own charge produce an out word pressure . This pressure is known as electrostatic pressure. In order to find the expression for this pressure .let us considor a charged surface .The electric field out side this charge sueface is .

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{............(1)}$$

where σ is the surface charge density .Due to this electric field E, the elementry area dA produce a force.

$$F = dqE \qquad \text{............(2)}$$

where dq is a charge in area dA which is given by

$$dq = \sigma\,dA \qquad \text{............(3)}$$

Therefore we have

$$F = \sigma\,d\,A.\,\sigma/2\varepsilon_0$$

$$F = \frac{\sigma^2}{2\varepsilon_0}dA$$

But the force per unit area is the out word preessuere , therefore

$$\boxed{p = \frac{\sigma^2}{2\varepsilon_0}}$$

It is the exepression for the pressure on the charged surface

Q.6. Write short notes on multipole . (2005,2011)

OR

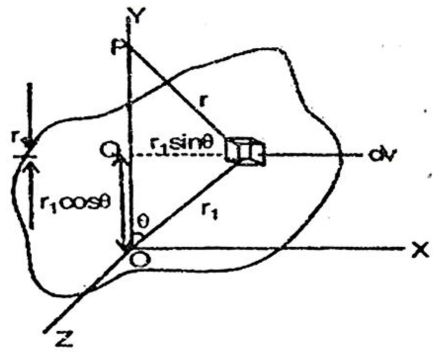
Derive an expression for the potential at a point due to an arbitrary charge distribution.

Sol. Multipole is a collective term for certain point charge system, which in the order of increasing complexity. A monpole has one pole, similarly a dipole has two poles and a quard pole has four poles etc. In general the number of pole in a multipole is always 2^n . Where n is the known order of miltipole and it is a possitive 'integer. Thus for

n = 0, there is a monopole , n = 1, there is a dipole

n = 2, there is a quadro pole

We have seen that the potential due to mono-pole varies 1/r , the potential due to dipole varios as $1/r^2$ and so on. Therefore is general it may be written that the potential various as $\frac{1}{r^{n+1}}$, where n is the order of multipole .



Now we devlop a general experssion for the potential of an orbitrary charge distribution , in power of 1/r.

Consider an arbitrary distribution of charge is the neighbour hood of orgin as shown in the folllowing figure.

The electric potential at the point P on the y -axis is

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho dv}{r} \qquad (1)$$

Where ρ is the volume charge density , and r is the distance of the charge from point P. Using law of cosine , we get,

$$r^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta \quad \text{or} \quad r^2 = r_1^2\left[1+\left(\frac{r_1}{r_2}\right)^2 - 2\left(\frac{r_2}{r_1}\right)\cos\theta\right]$$

$$r^2 = r_1^2\left[1+\frac{r_2}{r_1}\left(\frac{r_2}{r_1} - 2\cos\theta\right)\right]$$

$$r^2 = r_1^2(1+\varepsilon)$$

$$r = r_1(1+\varepsilon)^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where $\varepsilon = \left(\frac{r_2}{r_1}\right)\left(\frac{r_2}{r_1} - 2\cos 0\right)$

Now from eqn (2) we get -

$$\frac{1}{r} = \frac{1}{r_1}(1+\varepsilon)^{-\frac{1}{2}}$$

Using Binomial expansion , we get

$$\frac{1}{r} = \frac{1}{r_1}\left(1 - \frac{1}{2}\varepsilon + \frac{3}{8}\varepsilon^2 ------\right)$$

$$\frac{1}{r}=\frac{1}{r_1}\left[1-\frac{1}{2}\left(\frac{r_2}{r_1}\right)\left(\frac{r_2}{r_1}-2\cos\theta\right)+\frac{3}{8}*\left(\frac{r_2}{r_1}\right)^2\left(\frac{r_2}{r_1}-2\cos\theta\right)^2 ------\right]$$

The above summation can be written in term of legendre polynomial as

$$\frac{1}{r}=\frac{1}{r_1}\sum_{n=0}^{\infty}\left(\frac{r_2}{r_1}\right)^n P_n \cos\theta$$

Substituting these value in eqn (1) , then we get,

$$V=\frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\left(\frac{1}{(r_1)^{n+1}}\right)\int (r_2)^n P_n \cos\theta$$

It is the experssion of multipole .

Q. 7. What happens to the radius of a roap bubble .When it is negativily charged . Explain the answer ? **(2004, 06)**

Sol. Consider a roap bubble of radious r.In equilbrium, the excess pressure p acting out ward is balanced by inward pressure 4T/r resutling from surface tension .If bubble is gently charged , its surface is subjected to an additional out word pressure which makes the bubble to expend and the new equilbrium is established when the out word pressure is balanced by the in word pressure orgenating from the surface tension .If R be the new radius of the bubble , then the electric pressure is

$$p_E=\frac{\sigma^2}{2\varepsilon_0}=\frac{1}{2\varepsilon_0}\left(\frac{q}{4\pi R^2}\right)^2=\frac{q^2}{32\varepsilon_0\pi^2 R^4}$$

If p_1 is the new value of the excess pressure , then in equilbrium

$$p_1+p_E=\frac{4t}{r}$$

or

$$P_1+\frac{q^2}{32\varepsilon_0\pi^2 R^4}=\frac{4T}{r}$$

If the pressure inside the bubble becomes equal to the atmospheric pressure , then $p_1=0$ .The charge q_0 needed to obtain this condition is given

by $\frac{q_0^2}{32\varepsilon_0\pi^2 R^4}=\frac{4T}{r}$

The potential of the bubble in this condition is given by --

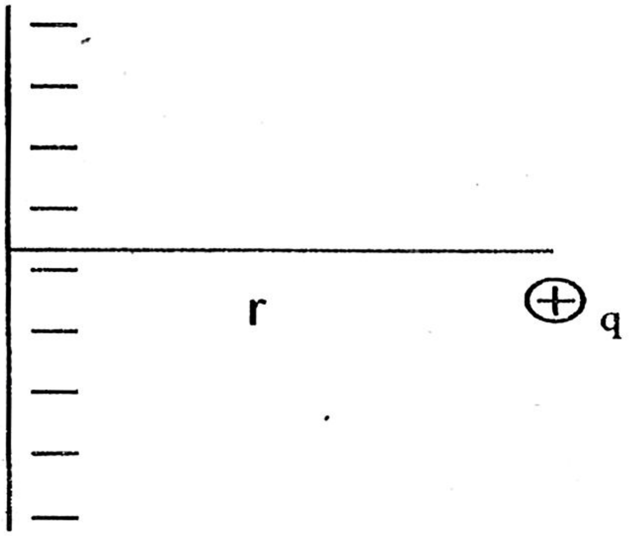
$$\boxed{V=\frac{q}{4\pi\varepsilon_0 R}=\sqrt{\frac{8RT}{\varepsilon_0}}}$$

Ques 8 :Write short notes on electrical images. **(GKP-2006,2012,2016)**

Soln. : If two equal and oppostite point charges are seperated by a certain distance then a plane passing through the mid point of the line joining them and perpendicular to this line is an equipotential surface. The concept of equipotential surface is useful in solving few porblem involving charges and conducting srufaces. It is best explained in the method of image.

यदि दो बराबर व विपरीत बिन्दु आवेशों को निश्चित दूरी पर रखा जय तो दोनों को जोड़ने वाली रेखा के मध्यबिन्दु से गुजरने वाली लम्बवत तल समविभव पृष्ठ होता है। समविभव पृष्ठ होता है। समविभव प्रश्नों को हल करने में उपयोगी हे। यह मैथड ऑफ इमेज में अच्छे से वर्णित है।

A charge + q. placed at a distance r from an infinitely large conducting plate experience a force, because of the induced charges on the conducting plate.



To evaluate this force, we must know the distribution of induced charge on the conducting plate. Suppose, we place an equal and opposite charge - q on the other side of the conducting plate at an equal distance r from the plate.

एक आवेश से दूरी पर स्थित एक अपरिमित लम्बाई का चालक प्लेट प्रेरित आवेशके लम्बाई के कारण एक बल का अनुभव करता है। इस बल को ज्ञात करने के लिए हमें प्रेरित आवेशों का चालक प्लेट पर वितरण ज्ञात होना चाहिए। माना एक दूसरा बराबर किन्तु विपरीत आवेश चालक प्लेट के दूसरी तरफ बराबर दूरी पर स्थित है।

Since every point on the ocnduction plate is equidistance form the two charges, therefore it is an equipotential plane.

As seen by the point charge + q, the induced charge of the conduction plate produce exactly the same field as would a point charge - q placed at a distance 2r from charge + q. Hence the force between the conduction plate and charge + q is obtained by applying Coulomb law between the charges + q and - q. This force is given by.

चूंकि चालक प्लेट पर प्रत्येक बिन्दु दोनों आवेशों से बराबर दूरी पर स्थित है अतः यह एक समविभव पृष्ठ है।

यदि + q बिन्दु आवेश द्वारा देखा जाय तो चालक प्लेट के प्रेरित आवेशों द्वारा उत्पन्न विद्युत क्षेत्र, - q बिन्दु क्षेत्र के बराबर होगा। इस प्रकार आवेश तथा चालक प्लेट के बीच कार्यरत बल को + q तथा - q आवेशों के बीच 2 r दूरी पर लगने वाला बल होगा। यह बल होगा।

$$F = \frac{1}{4\pi \in_0} \frac{q^2}{(2r)^2}$$

The expression of force obtaining in this way is called Method of

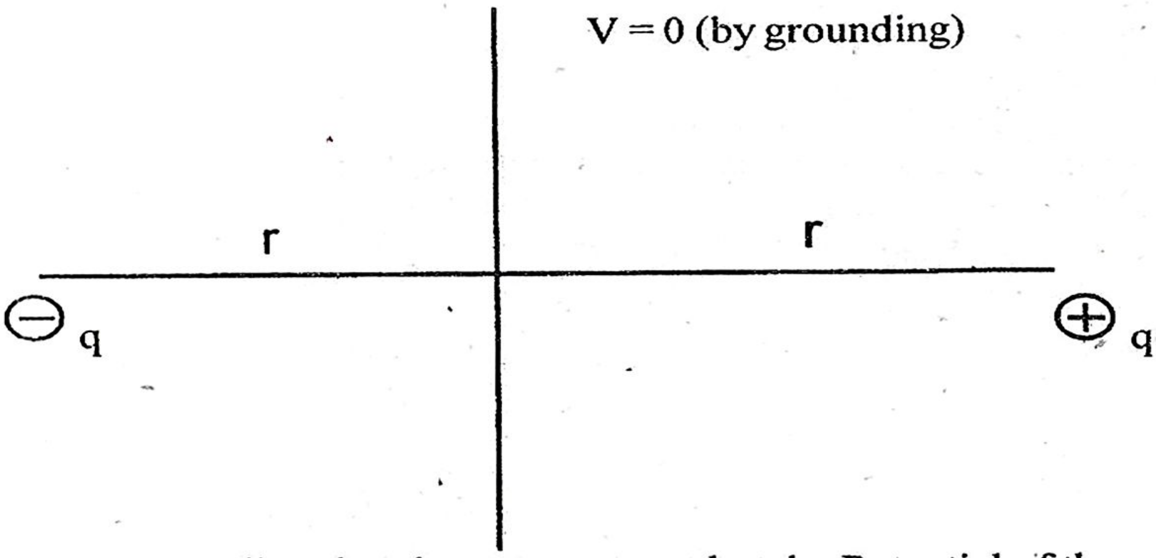


image. By grounding the plate, we ensure that the Potential of the conducting plate is kept constant and only induced charge contribute the force between the plate and the point chage.

इस प्रकार प्राप्त बल का एक्सप्रशन मैथड आफ इमेज कहलाता है। प्लेट को ग्राउण्ड कर चालक प्लेट का विभव को नियत पर सुनिश्चित किया जाता है और केवल प्रेरित आवेश ही प्लेट तथा बिन्दु आवेश के बीच बल प्रदान करती है।

Q.9 Obtain Coulomb's law from Gauss's theorem ? **(2013)**

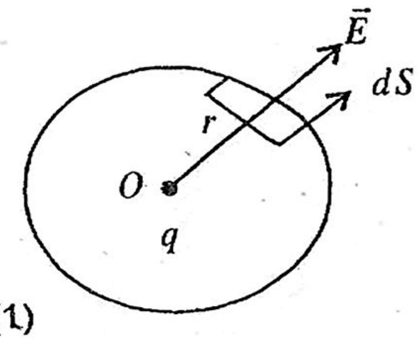
Sol. Let us consider an isolated positive point charge q and draw a Gausian sphere of radius r with q as centre. Both the electric field vector $\vec{E}$ and area vector $d\vec{S}$ are along the same direction i.e. the angle between them is 0^0 therefore - $\vec{E}.d\vec{S} = Ed\cos 0 = EdS$

Here the flux through this sphere is given by

$$\phi_E = \oint \vec{E}.d\vec{S} = \oint EdS$$

since E is constant for all patches it may be put outside the integral. Thus

$$\phi_E = E\oint dS = E(4\pi r^2)$$



Uses Gauss's Law, we get

$$\frac{q}{\epsilon_0} = E(4\pi r^2)$$

$$\Rightarrow \qquad E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \qquad \text{.........(1)}$$

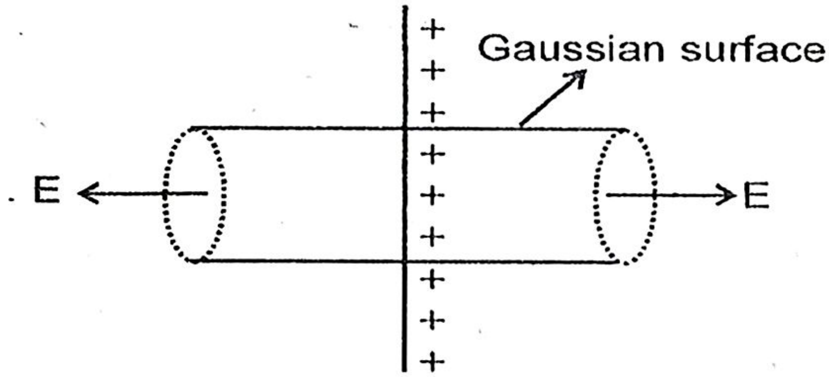
This equation gives the magnitude of electric field at a point at a distance r form point charge q . if we put a test charge q_0 at that point, then the force on the test charge is -

$$F = E.q_0 = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$$

This is Coulomb's law. Thus the Gausi's law is equivalent to Coulomb's law and serves equally well as the basic law of electrostatics.

Q. 10 Find the electric field intensity at a point in the vicinity of an infinite sheet of charge. **(2014)**

Sol. Consider an infinite plane with uniform surface charage density is σ constract a cylinderical gaussian surface, half of which lies on one side of the plane and rest on the other side. The electric Flux through the curved surface of the gaussian surface is zero.



If A is the area of the flat surface of the gaussian surface, then the total charge enclosed within it is $q = \sigma.A$.

Now applying Gauss's theorem, we get -

$$EA + EA = \frac{\sigma A}{\in}$$

or, $$E = \frac{\sigma}{2 \in_0}$$

For positive charge on the plane, the electric field points away from the plane and for negative charge it points towards the plane.

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