# 3 Forced Oscillations

**Q.1 : What do you understand by forced vibration ? Establish equation for forced vibration of a system and discuss sharpens of resonance.**
**(1999, 2003, 2007)**

**OR**

**Solve the equation of motion of a damped harmonic oscillator in presence of an external harmonic force. Discuss dependence of average power, absorbed by the oscillator on the frequency of the external harmonic oscillator.** **(GKP-2009,2011,2012,2016)**

**OR**

**What is meant by sharpness of resonance.** **(GKP-2013)(SU-2016)**

**OR**

**A particle move under the influence of an external periodic force. Set up the differential equation of motion the system and obtain its steady state solution.**
**(2014)**

**Sol$^n$ :** When a body capable of oscillation is subjected by an external force then it begins to oscillate under the action of the applied force. In the beginning, the body tries to oscillate with its natural frequency while the external force tries to impose its own frequency upon the body. Thus there is a soft of tussal between the external force and the drive during which the amplitude of oscillation rises and falls irregularly a number of times. Finally the body yield to the external force and oscillates with '**constant**' amplitude and '**with**' the frequency of the force. Its oscillation are then called '**forced oscillation**' the oscillating body is called **driven hormonic oscillator** and the external force is **driving force**.

जब किसी दोलन करने वाले पिण्ड पर कोई वाह्य बल लगाया जाय तब यह पिण्ड इस वाह्य बल के कारण दालन करने लगता है। प्रारम्भ में पिण्ड अपनी स्वाभाविक आवृत्ति के अनुसार दोलन करता है जबकि बाह्य बल अपनी स्वाभाविक आवृत्ति पिण्ड पर आरोपित करने की कोशिश करता है। अतः वहां वाह्य बल तथा प्रेरक के बीच खिंचाव पैदा हो जाता है जिससे दोलन का आयाम अनियमित ढ़ंग से बढ़ता और घटता है। अंततः पिण्ड वाह्य बल के प्रभाव में नियत आयाम तथा वाह्य बल की आवृत्ति के अनुसार दोलन करता है। तब यह दोलन **प्रणोदित दोलन** कहलाता है। दोलनी पिण्ड को प्रेरित आवर्ती दोलक तथा वाह्य बल को प्रेरक बल कहते हैं।

**Equation Of Forced Vibration-** Consider a system, oscillating about an equilibrium position under an external periodic force. Let $x$ be its displacement from the equilibrium position at an instant during the oscillation its instantaneous velocity is $dx/dt$. The force acting upon the system at this instant are :

(i) A restoring force proportional to the displacment but acting in the oppsite direction i.e. $-kx$, where $k$ is the force constant.

(ii) A frictional force proportonal to the velocity but acting in the opposite direction. This may written as :- $-r\dfrac{dx}{dt}$

(iii) An external periodic force represent by $F_0 \sin pt$. Where $F_0$ is the maximum value of this force and $p$ is its angular frequency.

Thus the total force acting on the system is

$$F = -kx - r\frac{dx}{dt} + F_0 \sin pt$$

Thus the equation of motion of the oscillation

$$m\frac{d^2x}{dt^2} = -kx - r\frac{dx}{dt} + F_0 \sin pt$$

or
$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dy}{dx} + \frac{k}{m}x = \frac{F_0}{m}\sin pt$$

Putting $\frac{r}{m} = 2R$, $\frac{k}{m} = \omega^2$ and $\frac{F_0}{m} = f_0$ in the above equation,

we get
$$\frac{d^2x}{dt^2} + 2R\frac{dx}{dt} + \omega^2 x = f_0 \sin pt \qquad \text{.............(1)}$$

It is the differential equation of motion of forced hormonic oscillator. Let the solution of this equation is

$$x = A\sin(pt - \theta) \qquad \text{..............(2)}$$

Where $A$ and $\theta$ are arbitray constant.

Differntiating eqn (2) twice with respect to '$t$', then we get

$$\frac{dx}{dt} = Ap\cos(pt - \theta)$$

and
$$\frac{d^2x}{dt^2} = -Ap^2 \sin(pt - \theta)$$

Putting these values in equation (1) we get

$$-p^2 A\sin(pt - \theta) + 2rpA\cos(pt - \theta) + \omega^2 A\sin(pt - \theta) = f_0 \sin\{(pt - \theta) + \theta\}$$

or, $A(\omega^2 - p^2)\sin(pt - \theta) + 2rpA\cos(pt - \theta)$

$$= f_0 \sin(pt - \theta)\cos\theta + f_0\cos(pt - \theta)\sin\theta$$

If this equation is to be satisfied for all values of $t$, then the coeffieicient of $\sin(pt - \theta)$ and $\cos(pt - \theta)$ on the two sides must be equal. That is

$$A(\omega^2 - p^2) = f_0\cos\theta \qquad \text{.................(3)}$$

$$2rpA = f_0\sin\theta \qquad \text{.................(4)}$$

squaring and adding equation (3) and (4), we get

$$A^2\{(\omega^2 - p^2)^2 + 4r^2p^2\} = f_0^2$$

or
$$A = \frac{f_0}{\sqrt{\{(\omega^2 - p^2)^2 + 4r^2p^2\}}} \qquad \text{..............(5)}$$

Dividing eq$^n$(4) by eqn(3) we get

$$\tan\theta = \frac{2rp}{\omega^2 - p^2} \qquad \text{...............(6)}$$

putting the value of A in equation (2) we, get

$$y = \frac{f_0}{\sqrt{\{(\omega^2 - p^2)^2 + 4r^2p^2\}}}\sin(pt - \theta)$$

It is the solutionof the differential equation of the forced hormonic oscillator.

**2. प्रणोदित कम्पन का समीकरण** –माना एक निकाय किसी साम्यावस्था के इर्द गिर्द किसी वाह्य बल के कारण दोलन करता है। माना दोलन के दौरान किसी क्षण $t$ पर इसका साम्यावस्था से विस्थापन $x$ है तो तत्कालिक वेग $\frac{dx}{dt}$ होगा। इस क्षण निकाय पर कार्यरत बल है

3. एक प्रत्यानयन बल जो कि विस्थापन के समानुपाती तथा इसकी विपरीत दिशा में होगा $ie - kx$ जहां $k$ एक नियतांक है।

4. एक घर्षण बल जो कि वेग के समानुपाती तथा इसकी विपरीत दिशा में होगा। एक वाह्य आवर्त बल $F_0 \sin pt$ जहां .इस बल का महत्तम मान है तथा $p$ इसकी कोणीय आवृत्ति है।

अतः निकाय पर कार्यरत कुल बल है $F = -kx - r\frac{dx}{dt} + F_0 \sin pt$

अतः दोलनी गति का समीकरण होगा

$$m\frac{d^2x}{dt^2} = -kx - r\frac{dx}{dt} + F_0 \sin pt$$

अथवा
$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin pt$$

$\frac{r}{m} = 2R$ तथा $\frac{k}{m} = \omega^2$ तथा $\frac{F_0}{m} = f_0$ रखने पर

उपयुक्त समीकरण निम्न होगा–

$$\frac{d^2x}{dt^2} + 2R\frac{dx}{dt} + \omega^2 x = f_0 \sin pt$$

5. यह प्रणोदित आवर्ती दोलक की गति का अवकलन समीकरण है

माना इस समीकरण का हल है

$x = A\sin(pt - \theta)$ जहां $A$ .तथा $\theta$ निरपेक्ष नियतांक है।

समीकरण 2 को $t$ के सापेक्ष अवकलित करने पर

$$\frac{dx}{dt} = Ap\cos(pt - \theta) \qquad \frac{d^2x}{dt^2} = -Ap^2 \sin(pt - \theta)$$

इन मानों को समीकरण (1) में रखने पर

$$-p^2 A\sin(pt - \theta) + 2rpA\cos(pt - \theta) + \omega^2 A\sin(pt - \theta) = f_0 \sin\{(pt - \theta) + \theta\}$$

or, $A(\omega^2 - p^2)\sin(pt - \theta) + 2rpA\cos(pt - \theta)$

$$= f_0 \sin(pt - \theta)\cos\theta + f_0 \cos(pt - \theta)\sin\theta$$

6. यदि यह समीकरण $t$ के समस्त मानों को संतुष्ट करे तो. $\sin(pt - \theta)$ तथा $\cos(pt - \theta)$ के गुणांक दोनों पक्ष में बराबर होंगे अर्थात

$$A(\omega^2 - p^2) = f_0 \cos\theta \qquad \text{............(3)}$$

$$2rpA = f_0 \sin\theta \qquad \text{............(4)}$$

समीकरण 3 तथा 4 को वर्ग करके जोड़ने पर

$$A^2\{(\omega^2-p^2)^2+4r^2p^2\}=f_0{}^2 \qquad ............(5)$$

$$A=\frac{f_0}{\sqrt{\{(\omega^2-p^2)^2+4r^2p^2\}}}$$

7. यह प्रणोदित अवार्ती दोलक की गति के अवकलन समीकरण का हल है।

$$y=\frac{f_0}{\sqrt{\{(\omega^2-p^2)^2+4r^2p^2\}}}\sin(pt-\theta)$$

**Sharpness Of Resonance** :- The quality factor of an oscillator is usually defined in term of energy loss as well as in term of the sharpness of resonance. That is by the relation

**अनुनाद की तीक्ष्णता:**— किसी दोलक के क्वालिटी फैक्टर को या तो उर्जा क्षय में या अनुनाद की तीक्ष्णता के पदो में व्यक्त करते हैं

$$Q=\frac{\omega}{\omega_2-\omega_1}=\frac{\omega}{\text{Band width}} \qquad ......... (8)$$

Where $\omega$ is the resonance frequency and $\omega_1$ and $\omega_2$ are the values of the driving frequencies at which the power drops to half its maximum value. i.e.

जहां $\omega$ अनुनाद की आवृत्ति तथा $\omega_1$ तथा $\omega_2$ वह प्रेरित आवृत्तियां है जिन पर शक्ति अपनी अधिकतम मान के आधे के बराबर होता है।

$$P_{av}=\frac{1}{2}(P_{av})_{\max} \qquad ............ (9)$$

The sharpnes of resonence in terms of $Q$ can be stated as follows. If $Q$ is very high, then the resonence of the system is sharp and if it is low then the resonance is flat.

अतः अनुनाद की तीक्ष्णता को क्वालिटी फैक्टर के रूप में इस प्रकार व्यक्त करते हैं। यदि $Q$ बहुत ज्यादा है तब निकाय में अनुनाद तीक्ष्ण है तथा जब $Q$ कम है तो अनुनाद चौड़ा है। अतः अवशोषित औसत शक्ति

The average power absorbed is $P_{ave}=\dfrac{mf_0^2p^2r}{(\omega^2-p^2)^2+4r^2p^2}$

$$(P_{ave})_{\max}=\frac{mf_0^2}{4r} \qquad ............ (10)$$

Now from the relation (9), we get

$$\frac{mf_0^2p^2r}{(\omega^2-p^2)^2+4r^2p^2}=\frac{1}{2}\left(\frac{mf_0^2}{4r}\right)$$

or $\quad (\omega^2-p^2)^2+4r^2p^2=8r^2p^2$

or $\quad (\omega^2-p^2)^2=4r^2p^2$

or $\quad \omega^2-p^2=\pm 2rp$

or $$\omega - p = \pm \frac{2rp}{(\omega + p)}$$

If $\omega_2 > \omega_1$, then $$\omega_2 - p = + \frac{2rp}{(\omega + p)}$$

or, $$\omega_2 = p + \frac{2rp}{(\omega + p)}$$

If $\omega_2 < \omega_1$, then $$\omega_1 - p = - \frac{2rp}{(\omega + p)}$$

or, $$\omega_1 = p - \frac{2rp}{(\omega + p)}$$

subtracting, we get

$$\omega_2 - \omega_1 = \frac{4rp}{(\omega + p)}$$

If $\omega \approx p$, then $$\omega_2 - \omega_1 = 2r$$

$$\boxed{\omega_2 - \omega_1 = \frac{1}{\tau}}$$

where $\tau = \frac{1}{2r}$ is the relaxation time

**Quality Factor :** The quality factor of forced oscillator is defined as

$$Q = \frac{\omega}{\omega_2 - \omega_1}$$

the sharpness of resonance in terms of $Q$ can be stated as fallows :

(a) If band width $(\omega_2 - \omega_1)$ is small that is $Q$ is high, the resonance in the system is very sharp.

(b) If band with $(\omega_2 - \omega_1)$ is large that is $Q$ is low, the resonance is flat.

**Q.2: Write down the differential equation of motion of a forced mechanical oscillation? find the expresion for the displacement and the velocity of the system and down their phase relationship with driving force. (2001)**

**Sol$^n$:** For the differential equation of forced mechanical oscillation : See Q.N.-1

**Displacement Or Amplitude Resonance :** The amplitude of the forced oscillation depend on the difference between the driving frequency $p$ and the natural frequency $\omega$ of the oscillator.

**Case (i) :** When ( $p << \omega$ ) . At very low driving frequency is low , we have

$$A = \frac{f_0}{\omega^2} \approx \frac{f_0 / m}{(k / m)} = \frac{F_0}{k}$$

or, $$A = \frac{f}{\omega^2} \approx \frac{f_0 / m}{(k / m)} = \frac{F_0}{k}$$

This shows that the amplitude depend only on the force constant. It is indenpendent of the mass, of the damping and of the driving frequency.

यह समीकरण दिखाता है कि आयाम केवल बल नियतांक पर निर्भर करता है। यह द्रव्यमान से स्वतंत्र है।

**Case (ii) :** When $p \approx \omega$, that is the impressed and natural frequencies are equal.

$$A_{\max} = \frac{f_0}{2rw}$$

It is clear that for low damping (r is small), the amplitude is higher. If there is no damping ($r = 0$), the amplitude for this resonat vibration will be infinite. Actulally it never happens as $r$ is never zero.

In presence of damping, amplitude maximum does not occur exactly at $p = \omega$.

**Case (iii) :** At very high driving frequency ($p >> \omega$) we have

$$A = \frac{f}{p^2} \approx \frac{F_0}{mp^2}$$

This show that the amplitude, which now depends on the mass, continously decreases as the driving frequency is further increased.

यह समीकरण दिखाता है कि आयाम जो कि द्रव्यमान पर निर्भर करता है, क्रमशः घटता है जब प्रेरक वेग आवृत्ति बढ़ती है।

**Velocity Amplitiude -** When a system of mass $m$, having a natural angular frequency $\omega$ oscillates under an external force $F_0 \sin pt$, the as displacement of the forced oscillation at any instant $t$ is given by

**वेग आयाम:—** जब एक $m$ द्रव्यमान का निकाय जिसकी स्वाभाविक कोणीय आवृत्ति $\omega$ है, किसी वाह्य बल $F_0 \sin pt$ के कारण दोलन करता है तो किसी क्षण $t$ पर प्रणोदित दोलन

$$x = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} \sin(pt - \theta)$$

Therefore, the instantaneous velocity

$$v = \frac{dx}{dt} = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} \cos(pt - \theta)$$

or,

$$v = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} \sin\{pt - (\theta - \pi/2)\}$$

The Velocity will be maximum when $\sin\{pt - (\theta - \pi/2)\} = 1$. The maximum value of velocity is known as " **Velocity Amplitude**" and is denoted by $V_A$.

$$V_A = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}}$$

It is clear that velocity amplitude depends upon $\omega$ and $p$.

**Phase :** The displacement of forced oscillation lags behind the driving force $F_0 \sin pt$ by an angle $\theta$, which is given by $\tan\theta = \dfrac{2rp}{\omega^2 - p^2}$

From this equation it is clear that the phase difference $\theta$ depends upon damping $r$ and the difference of $\omega$ and $p$. Fallowing three case may be aries.

**Case (i) :** If $p < \omega$, $\tan\theta$ is +ve, $\theta$ lies between $0$ and $\pi/2$.

यदि $p < \omega$ तब $\tan\theta$ धनात्मक होगा। इस केस में $\theta$ का मान 0 से $\pi/2$ के बीच स्थित होगा।

In this case, $\tan\theta$ is small but positive, If $r$ is too small, $\theta$ will be nearly zeo. In this condition the forced oscillation will be almost in phase with the driving force.

**Case (ii) :** If $p=\omega$, $\tan\theta$ is $\infty$, $\theta=\pi/2$.

यदि $p=\omega$ तब $\tan\theta$ अनन्त होगा, इस केस में $\theta$ का मान $\pi/2$ होगा।

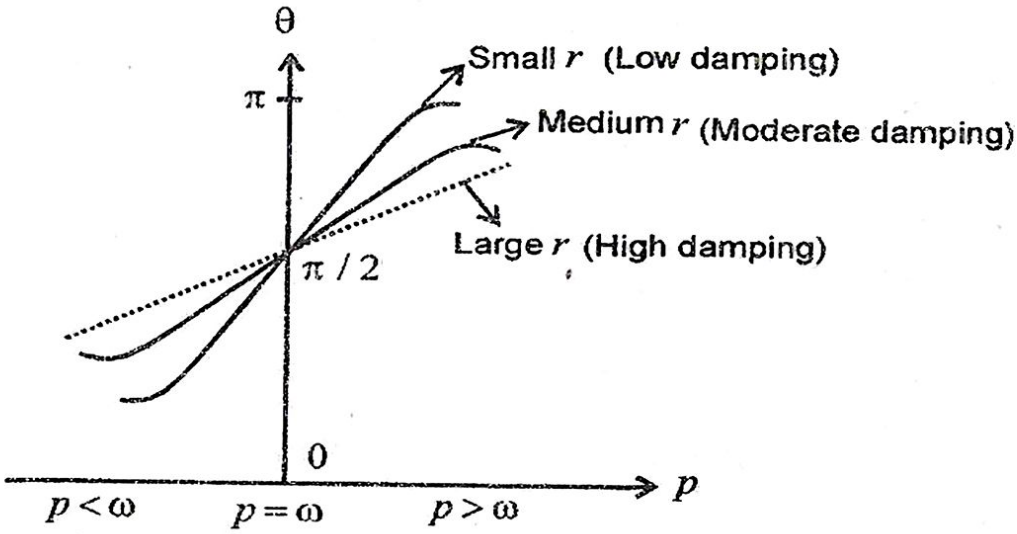
In this case $\tan\theta$ is $\infty$, $\theta=\pi/2$. At resonance, the oscillations always lag behind the force by $\pi/2$. It means at resonance the displacement is minimum when the driving force is maximum and vice versa.

**Case (iii) :** If $p>\omega$ $\tan\theta$ is - ve, $\theta$ lies between $\pi/2$ and $\pi$.

$p>\omega$ तब $\tan\theta$ ऋणात्मक होगा। इस केस में $\theta$ का मान $\pi/2$ तथा $\pi$ के बीच होगा।

In this case $\tan\theta$ is very small and negative, If $r$ is too small, $\theta$ will be almost opposite in pahse with the driving force.

The variation of $\theta$ with $p$ is shown in the following figure :-




**Q.3 : Define sharpness of resonance and band width show that sharpness of resonance depend upon damping? Give an example of sharp resonance. (GKP-1999,2010,2013,2015,2016)**

**OR**

**Establish the relation between band width and quality factor. (2000)**

**Sol$^n$ :** The quality factor of an oscillator is usually defined in term of energy loss as well as in term of the sharpness of power resonance that is by the relation.

$$Q=\frac{\omega}{\omega_2-\omega_1} \qquad \text{........ (1)}$$

Where $\omega$ is the resonance frequency and $\omega_1$ and $\omega_2$ are the value of the driving frequency at which the power drops to half its maximum valve that is

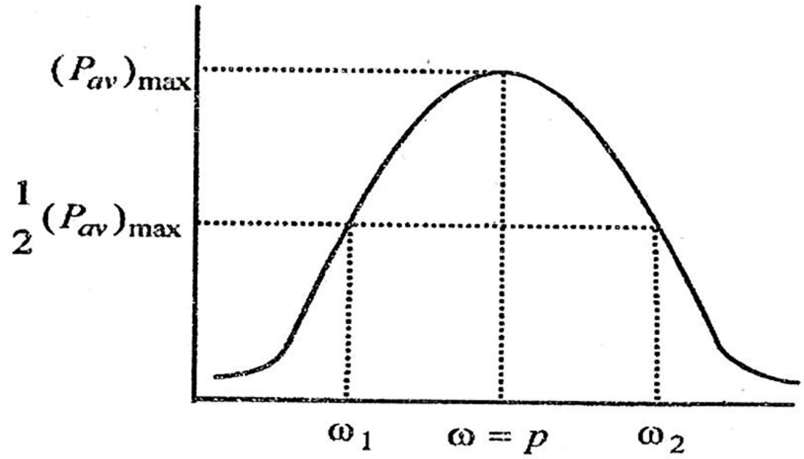
$$P_{av}=\frac{1}{2}(P_{av})_{\max} \qquad \text{........ (2)}$$

The difference of $\omega_1$ and $\omega_2$ is called the band width. The sharpness of resonance is mearsured by this quantity. thus the quality factor can be written as

$$Q=\frac{\omega}{\text{Band width}} \qquad \text{........ (3)}$$

The sharpness of resonance in term of $Q$ can be stated as follows.If $Q$ is very high then the resonance in the system is sharp and if it is low then the resonance is low.

The average power absorbed is $P_{ave} = \dfrac{mf_0^2 p^2 r}{(\omega^2 - p^2)^2 + 4r^2 p^2}$




$$(P_{ave})_{max} = \frac{mf_0^2}{4r}$$

Now from the relation (9), we get

$$\frac{mf_0^2 p^2 r}{(\omega^2 - p^2)^2 + 4r^2 p^2} = \frac{1}{2}\left(\frac{mf_0^2}{4r}\right)$$

or $\quad (\omega^2 - p^2)^2 + 4r^2 p^2 = 8r^2 p^2$

or $\quad (\omega^2 - p^2)^2 = 4r^2 p^2$

or $\quad \omega^2 - p^2 = \pm 2rp$

Since $\omega_1 < \omega$ and $\omega_2 > \omega$ we have

$$\omega^2 - \omega_1^2 = 2r\omega_1$$

and $\quad \omega^2 - \omega_2^2 = -2r\omega_2$

From these two equation, one get

$$\omega_2^2 - \omega_1^2 = 2r(\omega_1 + \omega_2)$$

or $\quad (\omega_2 - \omega_1)(\omega_2 + \omega_1) = 2r(\omega_2 + \omega_1)$

or $\quad (\omega_2 - \omega_1) = 2r$

But we know that $2r = k/m$, Where $k$ is force constant.

Thus, $\quad (\omega_2 - \omega_1) = k/m \quad$ ........ (5)

Thus the quality factor can be written as

$$Q = \frac{\omega}{\text{Band width}} = \frac{\omega}{k/m} = \frac{m\omega}{k} \quad \text{....... (6)}$$

It is evident from equation (5) and (6) that for low damping, bond width is small and Q is high and therefore the resonance is sharp. For large damping band width is large and Q is small and hence the resonace is flat.

**Example :-** An example of sharp resonance is found in case of sonometer wire in which the damping is small when the wire is timed exactly to a given tuning fork, it oscillates vigorously but even for very slight change from the correct length the oscillations practically vanish.

An example of flat resonance is found in case of a resononce column in which the oscillations of the air are subjected to a larger damping. when the length of the air-colmn is adjusted to match its frequency with the frequency of a given turning fork, the column responds over a wide range and it is difficult to find out the excat resonant length of the column.

**उदाहरण**— तीक्ष्ण अनुनाद का एक उदाहरण सोनोमीटर के केस में पाया जाता है। जिसमें मन्दन कम होती है जब तार दिये गये स्वरित्र द्विभुज से मेल खाता है। यह दोलन करता है परन्तु दोलक के वास्तविक लम्बाई से बहुत कम अन्तर पर ।

**Q.4 : What are Mechanical and Electrical Vibration. Distinguished between Mechanical and Electrcial impedance?**

**(1999,2001,2002, 2009,2012,2013,2015)**

**OR**

**Whate are the Mechanical and Electrical vibration. (2008)**

**Sol$^n$ : Mechanical System :** - It consist of a mass $m$ connected to one end of a spring of which the other end is connected to a wall. The force constant of the spring is $k$. Then the frequency of the system is given by

**यांत्रिकी निकायः** इस प्रकार के निकाय में किसी स्प्रिंग के एक सिरे पर $m$ द्रव्यमान जुड़ा होता है तथा स्प्रिंग का दूसरा सिरा एक दीवार से जोड दिया जाता है, यदि स्प्रिंग का बल नियतांक $k$ हो तो निकाय की दोलन की आवृत्ति निम्न सूत्र से दिया जायेगा।

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

**Electrical System :-** n electric analogue of the mechanical system consists of an inductance $L$ and a capacitance $C$ in series. An alternating voltage $E = E_0 \sin\omega t$ is applied to it. In a series resonant circuit, the frequency of the system is given by

**वैद्युत निकायः**— किसी वैद्युत निकाय में एक प्रेरकत्व तथा एक संधारित्र श्रेणी क्रम में जुड़े होते हैं। यदि एक प्रत्यावर्ती वोल्टेज $E = E_0 \sin\omega t$ को एक श्रेणीक्रम अनुनादी परिपथ में प्रवाहित किया जाय तो निकाय का आवृत्ति निम्न समीकरण से दिया जाता है

$$n = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right)}$$

From the above two expression it is clear that mass $m$ in mechanical system corresponds to inductance $L$ in electrical system (both express inertia of the system) while force constant $k$ in mechanical system corresponds to $1/C$ in electrical system. Mechanical and Electrical vibration both are damped due to frictional force and resistance respectively.

उपर्युक्त दोनों समीकरणों से स्पष्ट है कि यांत्रिकी निकाय में द्रव्यमान $m$ वैद्युत निकाय में प्रेरकत्व $L$ के समतुल्य है। दोनों ($m$ तथा $L$) निकाय की जड़त्व है जबकि बल नियतांक $k$ तथा $1/C$ क्रमशः यांत्रिकी निकाय तथा वैद्युत निकाय में एक दूसरे के समतुल्य है। दोनों निकाय यांत्रिक और वैद्युत निकाय अवमंदित है जो कि घर्षण बल तथा प्रतिरोध के कारण होता है।

**Electrical Impedance :-** This is defined as the ratio of the voltage and current or the voltage requried to produce a unit current in the ciruit. That is

**वैद्युत प्रतिबाधा**— किसी परिपथ में एकांक विद्युत धारा प्रवाहित करने के लिए आवश्यक

विभव को वैद्युत प्रतिरोध कहते हैं अर्थात् किसी परिपथ में विभव तथा धारा के अनुपात को विद्युत प्रतिबाधा कहा जाता है। अर्थात $Z_e = \frac{v}{i}$

**For a L-C-R circuit impedance is**

किसी L-C-R परिपथ के लिए प्रतिबाधा

$$Z_e = \sqrt{R^2 + \left(\omega_L - \frac{1}{\omega_c}\right)^2}$$

**Mechanical Impedance:-** This is defined as force required to produce a unit velocity in the oscillator. So

**यांत्रिकी प्रतिबाधा:–** किसी यांत्रिक दोलक में एकांक वेग उत्पन्न करने के लिए आवश्यक बल को यांत्रिक प्रतिबाधा कहते हैं। अर्थात

$$Z_m = \frac{F}{V}$$

The impendance of a mass spring system is

किसी द्रव्यमान स्प्रिंग का प्रतिबाधा

$$Z_e = \sqrt{r^2 + \left(\omega_m - \frac{k}{\omega}\right)^2}$$

Where $r$ is a proportionality constant.

जहां $r$ समानुपातिक नियतांक है।

**Q.5 : Explain resonance in a parallel resonant circuit why does this circuit called rejector ? (2002,2015)**

**Sol$^n$ :** Parallel resonant circuit consists of $R$,, $L$ and $C$ all in parallel, fig 1(a) and $L$ and $C$ in parallel fig 1(b)

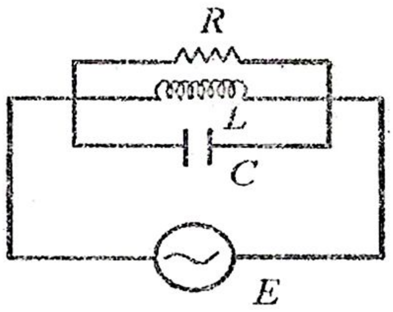



Fig - 1(a)

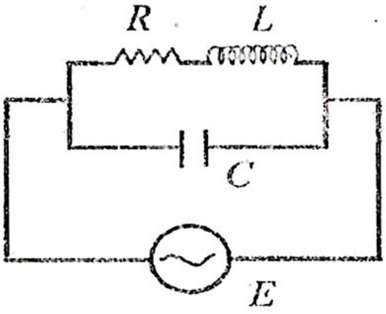



Fig - 1(b)

Pure LCR circuit is in resonante when

$$\omega C = \frac{1}{\omega L} \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

As in the series LCR circuit the resonant frequency is

$$n = 2\pi\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right)}$$

1. समान्तर अनुनादी परिपथ में $R, L$ तथा $C$ तीनों चित्र 1 $a$ अथवा चित्र 1 $b$ के अनुसार समान्तर में होते हैं। चित्र 1 $a$ में $L,C$ तथा $R$ तीनों समान्तर क्रम में है जबकि चित्र 1 $b$ में $L,C$ किसी शुद्ध $LCR$ परिपथ में अनुनाद के लिए

$$\omega = \frac{1}{\sqrt{LC}} \text{ होना चाहिए।}$$

जबकि श्रेणीक्रण $LCR$ परिपथ में अनुनादी आवृत्ति

$$n = 2\pi\frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} \text{ होता है।}$$

(ii) Let an alternating voltage $E_0 \sin\omega t$ is applied to a circuit containing a capacitance $C$ and inductance $L$ in parallel and resistance of inductance is much less than inductive reactance $\omega_L$. Let the maximum valve of alternating currents through capacitance and inductance are $i_c (= E_0 \omega c)$, and $i_L (= E / \omega_L)$ in capactance, the alternating current will lead by 90 in phase relative to emf. and in inductance the current will lag by $90^0$ in phase relative to emf. Therefore there will be a phase difference of $180^0$ between these currents and applied supply voltage will be equal to the difference of $i_c$ and $i_L$ .

For small values of frequency of e.m.f the value of $i_c$ will be small and the value of $i_L$ will be large. If the frequency of applied e.m.f. is increased then $i_c$ will increase and $i_L$ will decrease. The value of $i_c$ will be equal to $i_L$ when

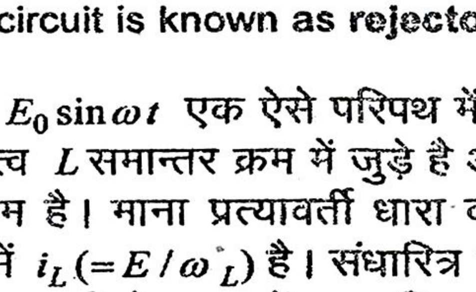
$$\omega L = \frac{1}{\omega c} \quad \text{or} \quad \omega = \frac{1}{\sqrt{Lc}}$$

and then applied frequency,

$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{Lc}}$$

The value of supply current in this state will be minimum but the valves of $i_c$ and $i_L$ will be large enough. As the value of supply current at resonance is a minimum, therefore this circuit is known as **rejector curcuit for this particular** frequency.

2. माना एक प्रत्यावर्ती धारा $E_0 \sin\omega t$ एक ऐसे परिपथ में प्रवाहित किया जाता है जिसमें एक संधारित्र $C$ तथा प्रेरकत्त्व $L$ समान्तर क्रम में जुड़े है और प्रेरकत्त्व का प्रतिरोध उसके प्रेरण प्रतिघात से काफी कम है। माना प्रत्यावर्ती धारा का अधिकतम मान संधारित्र में $i_c (= E_0 \omega c)$ तथा प्रेरकत्त्व में $i_L (= E / \omega_L)$ है। संधारित्र में प्रत्यावर्ती धारा विद्युत वाहक बल के सापेक्ष $90^0$ आगे होगा वही प्रेरकत्त्व में धारा वि0वा0बल के $90^0$ पीछे होगा। इस प्रकार धारा $i_c$ तथा $i_L$ में $180^0$ का कलान्तर होगा तथा आरोपित वोल्टेज $i_c$ तथा $i_L$ के अन्तर के बराबर होगा। विद्युत वाहक बल के छोटे मान की आवृत्ति के लिए $i_c$ का मान कम तथा $i_L$ का मान अधिक होगा। यदि आरोपित वि0वा0बल की आवृत्ति बढ़ाया जाय तो $i_c$ मे वृद्धि होगी तथा $i_L$ में कमी होगी। जब $i_c$ का मान $i_L$ के बराबर होगा तब इस अवस्था में आरोपित धारा न्यूनतम होगी लेकिन $i_c$ तथा $i_L$ का मान पर्याप्त अधिक होगा। चूंकि अनुनाद पर प्रवाहित धारा का मान न्यूनतम है, अतः इस प्रकार के परिपथ को इस अनुनादी आवृत्ति का **विरोधी परिपथ** कहते हैं।




$E(t) = E_0 \cos\omega t$

**Q.6 : In a series combination of inductance resistance and capacitance the half power points are obtained at frequency $f_1$ and $f_2$ respectively calulate the resonance frequency and phase angle at resonance. (2002)**

**Sol$^n$ :** The series combination of L-C-R circuit as shown in the figure.

In steady state the average input power is given by

$$\overline{P} = < E(t)I(t) >$$

or $$\overline{P} = < (E_0 \cos\omega t)\{I_0 \cos(\omega t - \theta)\} >$$

or, $$\overline{P} = E_0 I_0 < \cos\omega t(\cos\omega t \cos\theta - \sin\omega t \sin\theta) >$$

or, $$\overline{P} = \frac{1}{2} E_0 I_0 \cos\theta$$

or, $$\overline{P} = \frac{1}{2} E_0 \frac{E_0}{Z}.\frac{R}{Z} \qquad \left(\text{because } \cos\theta = \frac{R}{Z}\right)$$

$$\overline{P} = \frac{1}{2} E_0^2 \frac{R}{Z^2} = \frac{1}{2} E_0^2 \frac{R}{R^2 + (\omega L - 1/\omega C)^2}$$

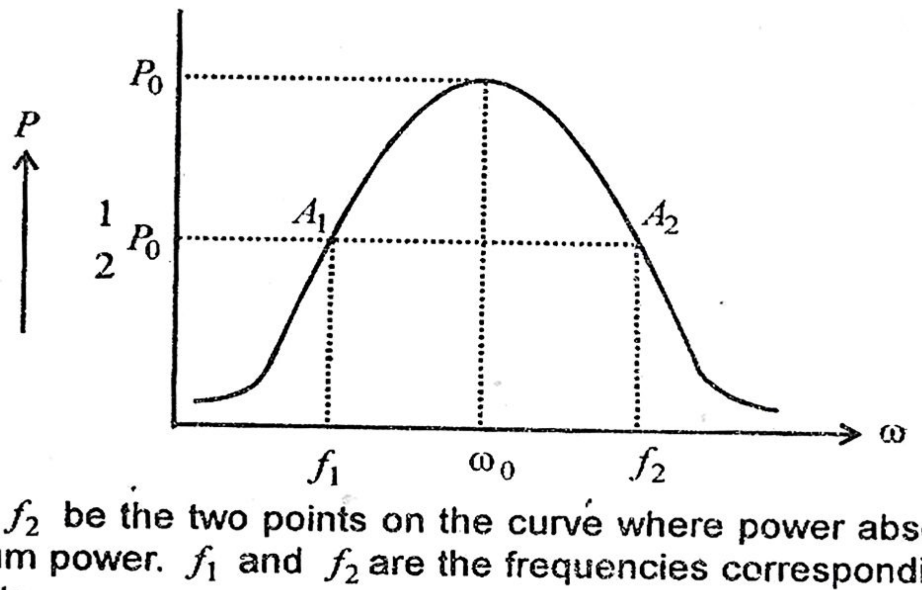
The average input power at resonance ($\omega = \omega_0$) is given by

$$\overline{P_0} = \frac{1}{2}\frac{E_0^2}{R}$$

In terms of $P_0$, the average input power is

$$\overline{P} = \overline{P_0}\frac{R^2}{R^2 + (\omega L - 1/\omega C)^2} \qquad \text{........ (1)}$$

The variation of average power $\overline{P}$ with frequency $\omega$ is shown in the following figure . $\overline{P}$ is maximum at frequency $\omega = \omega_0$ . It falls on both sides of the resonate frequency.




Let $f_1$ and $f_2$ be the two points on the curve where power absorption is half the maximum power. $f_1$ and $f_2$ are the frequencies corresponding to the half power points.

The frequencies $f_1$ and $f_2$ are the roots of equation (1) with $\overline{P}$ replaced by $\frac{1}{2}\overline{P_0}$ . i.e.

$$\frac{1}{2}\overline{P_0} = \overline{P_0}\frac{R^2}{R^2 + (\omega L - 1/\omega C)^2} \qquad R^2 + (\omega L - 1/\omega C)^2 = 2R^2$$

$$\omega L - 1/\omega C = \pm R \qquad \omega^2 - \omega_0^2 = \pm\frac{R}{L}\omega$$

since $f_1 < \omega_0$ and $f_2 > \omega_0$, we have

$$f_1^2 - \omega_0^2 = -\frac{R}{L}f_1 \qquad \text{......... (2)}$$

and $$f_2^2 - \omega_0^2 = \frac{R}{L}f_2 \qquad \text{........ (3)}$$

From equation (2) and (3), we get

$$f_2^2 - f_1^2 = \frac{R}{L}(f_1 + f_2)$$

$$\Rightarrow \qquad f_2 - f_1 = \frac{R}{L} \qquad \text{........(4)}$$

and eliminating $\frac{R}{L}$ from these equation,we get

$$f_1 f_2 = \omega_0^2$$

$$\Rightarrow \qquad \omega_0 = \sqrt{f_1 f_2} \qquad \text{........ (5)}$$

Thus the resonance frequency is the geometric mean of frequencies corresponding to half power points.At resonance, such a circuit, can give voltage amplification. At resonance the alternating potential at the ends of inductances and capacitance are equal in magnitude having a phase difference 180°. Therefore they neutralize each other and the applied e.m.f. is opposed by resistance Only in this state, voltage at the ends of inductance or capacitance may be increased in comparison to applied voltage.

इस प्रकार अनुनादी आवृत्ति अर्धशक्ति बिन्दुओं के सापेक्ष आवृत्तियों के गुणोत्तर माध्य के बराबर है। इस प्रकार का परिपथ अनुनाद पर वोल्टेज प्रवर्धन कर सकता है। अनुनाद की अवस्था मे प्रत्यावर्ती विभव, प्रेरकत्व तथा संधारित्र के सिरो पर परिमाण में बराबर होते हैं जिनके बीच 180° का कालान्तर पाया जाता है। अतः दोनों एक दूसरे को सामान्य कर देंगे तथा इस अवस्था में आरोपित वि0वा0बल केवल प्रतिरोध द्वारा रोधित होगा। इस अवस्था में संधारित्र तथा प्रेरकतव के सिरों का वोल्टेज, आरोपित वोल्टेज की तुलना में बढ़ाया जा सकता है।

**Q.7 : What is velocity resonance? Explain power absorption in a forced hormonic oscillation and obtain the formula for it? (2000)**

**Sol$^n$:** When a system of mass $m$, having a natural angular frequency $\omega$ oscillates under an external force $F_0 \sin pt$, the as displacement of the forced oscillation at any instant $t$ is given by

जब एक $m$ द्रव्यमान का निकाय जिसकी स्वाभाविक कोणीय आवृत्ति $\omega$ है, किसी वाह्य बल $F_0 \sin pt$ के कारण दोलन करता है तो किसी क्षण $t$ पर प्रणोदित दोलन

$$x = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}}\sin(pt - \theta)$$

Therefore, the instantaneous velocity

$$v = \frac{dx}{dt} = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2 p^2}} \cos(pt - \theta)$$

or, $$v = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2 p^2}} \sin\{pt - (\theta - \pi/2)\}$$

The Velocity will be maximum when $\sin\{pt - (\theta - \pi/2)\} = 1$. The maximum value of velocity is known as " **Velocity Amplitude**" and is denoted by $V_A$.

यह अधिकतम होगा जब $\sin\{pt - (\theta - \pi/2)\} = 1$ यह अधिकतम मान वेग आयाम कहलाती है।

$$V_A = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2 p^2}}$$

It is clear that velocity amplitude depends upon $\omega$ and $p$.

(i) When $p << \omega$ i.e., driving frequency is low, we may write

अत्यन्त छोटे प्रेरण आवृत्ति पर

$$V_A = \frac{f_0 p}{\omega^2} = \frac{(F_0 / m)p}{k / m} \approx \frac{F_0 P}{k}$$

Hence velocity amplitude depends upon force constant $k$.

(ii) When $p >> \omega$ i.e. , when driving frequency is high, we find

अत्यन्त उच्च प्रेरण आवृत्ति पर

$$V_A = \frac{f_0}{p} = \frac{F_0}{mp}$$

Thus velocity depends upon mass

(iii) When $p \approx \omega$ , the velocity amplitude is maximum, we have

$$V_A = \frac{f_0}{\sqrt{\left\{\left(\frac{\omega^2 - p^2}{p}\right)^2 + 4r^2\right\}}}$$

$$\left(\frac{\omega^2 - p^2}{p}\right)^2 = 0 \quad \text{or} \quad p = \omega$$

Thus, whatever be the damping, the velocity amplitude is maximum when the driving frequency is equal to the natural undamped frequency of the oscillator. This phenomenon is know as **Velocity Resonace.**

इस प्रकार किसी भी अवमंदन के लिए वेग आयाम तभी अधिकतम होगा जब प्रेरण आवृत्ति दोलक की प्राकृतिक आवृत्ति के बराबर होगी। यह घटना **वेग अनुनाद** कहलाता है।

**Power Absorption :-** In order to maintain steady state oscillation of the system, the driving force must supply energy equal to that lost in each cycle due to resistive force.Thus in steady state average power absorbed in a cycle is equal to the average power dissipated in a cycle.

**शक्ति अवशोषण:–** किसी निकाय की स्थिर अवस्था मे दोलन को बनाये रखने के लिए

प्रेरित बल द्वारा प्रत्येक चक्र में प्रतिरोधी बल के कारण कम हुए उर्जा के बराबर उर्जा का स्थानान्तरण (निकाय को) आवश्यक है। इस प्रकार स्थिर दोलन के प्रत्येक चक्र में औसत शक्ति अवशोषण उस चक्र में उर्जा क्षंय के बराबर होता है।

The instantenous power P absorbed by the oscillator is equal to the product of the instantenous driving force and the instantenous velocity.

दोलक द्वारा अवशोषित तात्कालिक शक्ति $P$ तात्कालिक प्रेरित बल तथा तात्कालिक वेग की गुणन के बराबर होता है।

that is
$$P = (F_0 \sin pt)\left(\frac{dx}{dt}\right)$$

or,
$$P = \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} \sin pt[\cos(pt - \theta)]$$

or,
$$P = \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} \sin pt[\cos pt \cos\theta + \sin pt \sin\theta]$$

$$P = \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4R^2p^2}} [\sin pt \cos pt \cos\theta + \sin^2 pt \sin\theta]$$

The average input power in a cycle is given by -

$$P_{av} = \frac{1}{T} \int_0^T P(t)dt$$

$$P_{av} = \frac{1}{T}\int_0^T \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} [\sin pt \cos pt \cos\theta + \sin^2 pt \sin\theta]\, dt \qquad \text{...... (4)}$$

But we know that for one time period

$$\frac{1}{T}\int_0^T \sin pt \cos pt\, dt = 0 \quad \text{and} \quad \frac{1}{T}\int_0^T \sin^2 pt = \frac{1}{2}$$

Hence, the average power absorb is

$$P_{av} = \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} \left(\frac{1}{2}\sin\theta\right) \qquad \text{...... (5)}$$

As we know that -

$$\tan\theta = \frac{2rp}{(\omega^2 - p^2)}$$

therefore,
$$\sin\theta = \frac{2rp}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}}$$

Substiting the value of $\sin\theta$ in equation (5) then we get -

$$P_{av} = \frac{mf_0^2 p^2 r}{\sqrt{\left(\omega^2 - p^2\right)^2 + 4r^2 p^2}}$$

It is the expression for the power absorb which is use in doing the work aganist the damping force.

**Q. 8 : Write short notes on the Series and Parallel resonance circuit?**

**(2001,2009)**

**Sol$^n$ : (a) Series Resorance Circuit:-** Alternating voltage $E = E_0 \sin\omega t$ is applied to a circuit having an inductance $L$, capacitance $C$ and resistance $R$ all in series. At any instant, the current through the $L$, $C$ and $R$ is the same.

**श्रेणीक्रम अनुनादी परिपथ:--** एक ऐसे परिपथ जिसमें एक प्रेरकत्व $L$ एक धारित्र $C$ तथा एक प्रतिरोध $R$ श्रेणीक्रम में जुड़े है, में एक प्रत्यावर्ती वोल्टेज $E = E_0 \sin\omega t$ प्रवाहित किया गया है।

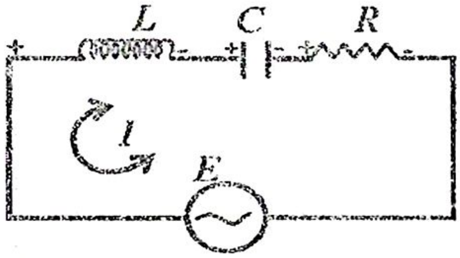
The effective resistance or impedance $Z$ of the circuit is given by

परिपथ में कार्यरत प्रतिरोध या प्रतिबाधा

$$Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}$$

The impotance $Z$ of the circuit is a minimum when $X_L = X_c$ and is equal to the resistance $R$ in the circuit.

इस परिपथ में प्रतिबाधा $Z$ न्यूनतम होने के लिए $X_L = X_c$ होना चाहिए जो कि प्रतिरोध $R$ के बराबर है।




$E(t) = E_0 \cos\omega t$

The maximum current flow through the circuit, which is in phase with the applied voltage. In this condition the circuit is called the **Series Resonance Circuit (or Acceptor Circuit)** and the resonant frequency is given by

परिपथ मे बहने वाला अधिकतम धारा आरोपित वोल्टेज के कला में होगा। इस अवस्था में परिपथ श्रेणीक्रम अनुनादी परिपथ (स्वीकार्य परिपथ) कहलाता है तथा अनुनादी आवृत्ति निम्न सूत्र द्वारा दिया जाता है।

$$X_L = X_c \quad \text{or} \quad L\omega = \frac{1}{\omega C} \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

$$n = \frac{\omega}{2\pi} \Rightarrow n = \frac{1}{2\pi\sqrt{LC}}$$

But this is the frequency of oscillatory discharge of a circuit containing capacitance $C$, inducatance $L$ and resistance $R$ in series. Therefore the value of alternating current in above circuit is maximum when the frequency of the applied emf is equal to the natural frequency of the circuit. It is the condition of electric oscillation.

परन्तु यह आवृत्ति, एक ऐसे परिपथ का दोलनी अनावेशन की आवृत्ति है जिसमे संधारित्र $C$ प्रेरकत्व $L$ तथा प्रतिरोध $R$ श्रेणी क्रम में है। अतः प्रत्यावर्ती धारा का मान उपर्युक्त परिपथ मे अधिकतम होने के लिए आरोपित वि0व0 बल की आवृत्ति परिपथ के स्वाभाविक आवृत्ति के बराबर होनी चाहिए।

Let the voltage across inductance and capacitance be $V_L$ and $V_C$ which are equal (at resonance), then

यह वैद्युत दोलन के लिए शर्त है। माना प्रेरकत्व तथा संधारित्र के सिरों पर विभवान्तर क्रमशः $V_L$ तथा $V_C$ है जो कि अनुनाद की अवस्था में एक दूसरे के बराबर है। तब

$$\frac{V_L}{V_{res}} = \frac{V_C}{V_{res}} = \sqrt{\frac{V_L}{V_{res}} \times \frac{V_L}{V_{res}}} = \sqrt{\frac{\omega L \times i_{res}}{V_{res}} \times \frac{(1/\omega C) \times i_{res}}{V_{res}}}$$

$$= \frac{i_{res}}{V_{res}}\sqrt{\frac{L}{C}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$\frac{V_L}{V_{res}}$ or $\frac{V_C}{V_{res}}$ representing the voltage amplituide at resonace is know as $Q$ - **factor** of the circuit.

$\frac{V_L}{V_{res}}$ या $\frac{V_C}{V_{res}}$ अनुनाद पर वोल्टेज आयाम प्रदर्शित करता हैं जिसे हम परिपथ का $Q$ - गुणज कहते हैं।

Thus, $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Taking the proper values of $L$, $C$ and $R$, $Q$ -factor may be increased.

$L$, $C$ and $R$ के उचित मानों का चुनाव कर हम $Q$ गुणज बढ़ा सकते हैं।

**(b) Parallel Resorance Circuit :** See question no. 5

**Q.9 : A forced harmonic oscillator has equal amplitudes at frequencies $\omega_1$ and $\omega_2$. Show that the frequency for amplitude resonance $\omega_r$ is given as**

$$\omega_r = \sqrt{\frac{1}{2}(\omega_1^2 + \omega_1^2)}$$ **(GKP-2016)**

**Sol^n^:** The amplitude of a forced oscillator is

$$A = \frac{F_0}{m\sqrt{(\omega^2 - p^2)^2 + 4r^2 p^2}} \quad \text{.........(1)}$$

$$A^2 = \frac{F_0^2}{m^2(\omega^2 - p^2)^2 + 4r^2 p^2}$$

$$A^2 = \frac{F_0^2 / m^2}{\omega^4 + p^4 - 2\omega^2 p^2 + 4r^2 p^2}$$

This equation can be arranged in to a quadratic equation in $p^2$ i.e.

$$p^4 - 2\omega^2 p^2 + 4r^2 p^2 + \omega^4 = \frac{F_0^2}{m^2 A^2}$$

$$p^4 - p^2(2\omega^2 - 4r^2) + \left(\omega^4 - \frac{F_0^{\ 2}}{m^2\ A^2}\right) = 0$$

For constant amplitude, this equation has two roots $\omega_1^{\ 2}$ and $\omega_2^{\ 2}$ .the sum of roots are $\omega_1^{\ 2} + \omega_2^{\ 2} = 2\omega^2 - 4r^2$

or $\omega_1^{\ 2} + \omega_2^{\ 2} = 2(\omega^2 - 2r^2)$

For a amplitude resonance $r \to 0$

$$\omega_1^{\ 2} + \omega_2^{\ 2} = 2\omega_r^{\ 2}$$

$\Rightarrow$ $\omega_r = \sqrt{\frac{1}{2}(\omega_1^{\ 2} + \omega_2^{\ 2})}$ **Proved.**

**Q. 10 : Describe how is the phase of driving force is related to the oscillation of the driven system near resonance. What is the effect of damping on the phase relationship. (2006,2014)**

**Sol^n^.** The forced oscillations $x = A\sin(pt - \theta)$ lag behind the driving force $F_0 \sin pt$ by an angle

**प्रणोदित दोलन** $x = A\sin(pt - \theta)$ ड्राईविंग बल $F_0 \sin pt$ से कोण δ से पीछे चलता है जिसका सूत्र निम्न होता है–

$$\tan\theta = \frac{2rp}{\omega^2 - p^2} \quad ..................(1)$$

Where $r$ is the damping factor, $\omega$ is the natural frequency of oscillation and $p$ is the driving frequency. This shows that the phase difference $\theta$ depends on damping and the difference between $\omega$ and $p$.

Following three cases may be arise.

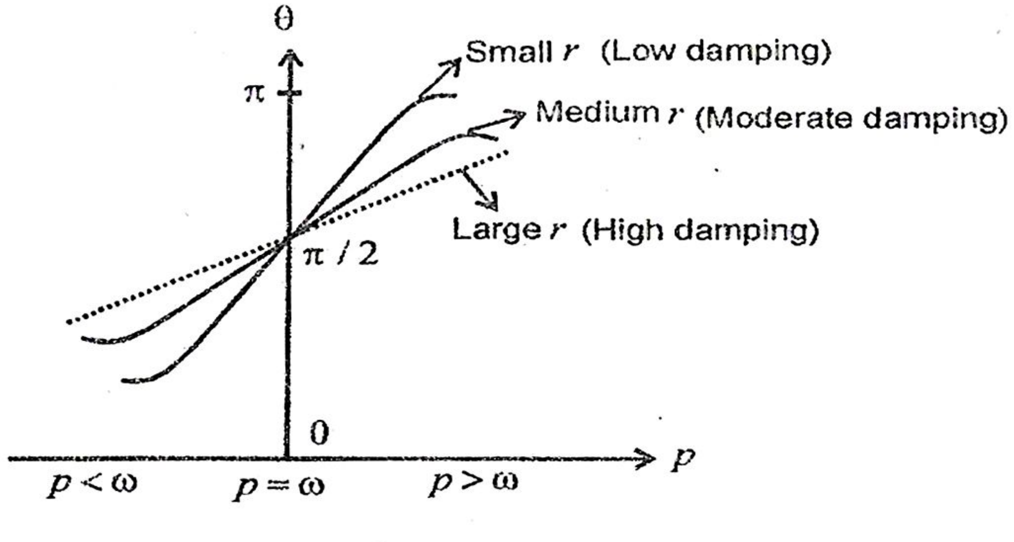
जहाँ $r$ डैम्पिंग फैक्टर है तथा $\omega$ दोलन की प्राकृतिक आवृत्ति है। $p$ ड्राईविंग आवृत्ति है। स्पष्ट है कि कलान्तर $\theta$ डैम्पिंग तथा $\omega$ और $p$ के बीच अन्तर पर निर्भर करता है। अतः निम्न तीन Cases पर विचार करते है।

**Case 1 :** If $p < \omega$ , $\tan\theta$ is positive $\theta$ lies between 0 and $\pi/2$

यदि $p < \omega$ , तथा $\tan\theta > 0$ तब $\theta$ का मान 0 तथा $\pi/2$ के बीच होता है।

**Case 2 :** If $p \approx \omega$ , $\tan\theta$ is infinite, $\theta = \pi/2$ .

यदि $p \approx \omega$ , तथा $\tan\theta = \infty$ तब $\theta = \pi/2$



Case 3 : If $p > \omega$, $\tan\theta$ is negative, $\theta$ lies between $\pi/2$ and $\pi$.

यदि $p > \omega$, तथा $\tan\theta < 0$ तब $\theta$ का मान $\pi/2$ तथा $\pi$ के बीच होता है।

Thus forced oscillations always lag behind the impressed force in phase by an angle between 0 and $\pi$, depending on the valve of $p$ relative to $\omega$. The variation of $\theta$ with $p$ are shown in the following figure.

अतः प्रणोदित दोलन $p$ के मान पर $\omega$ के सापेक्ष निर्भर करता है तथा आरोपित बल 0 से $\pi$ कोण तक पीछे रहता है। $\theta$ के सापेक्ष $p$ में होने वाले परिवर्तन निम्न चित्र के माध्यम से दर्शाया गया है।

**Effect of Damping on Phase :—** If damping is very small $r \approx 0$, then

$$\tan\theta = \frac{2p}{\omega^2 - p^2}$$

When $p < \omega$ $\tan\theta$ is very small but positive, so that $\theta$ is nearly zero. This means that under this condition the forced oscillations are practically in phase with driving force.

When $p > \omega$ $\tan\theta$ is very small but now negative, so that $\theta$ is very nearly $\pi$. Under this condition the forced oscillations are almost opposite in phase with the driving force.

When $p \approx \omega$, $\tan\theta = \infty$ so that $\theta = \pi/2$. Thus at resonance the oscillations always lag behind the force by $\pi/2$. This means that the displacement is minimum when the force is maximum and vice-versa.

**कला पर डैम्पिंग का प्रभाव :—** यदि डैम्पिग का मान काफी कम है तब

$$\tan\theta = \frac{2p}{\omega^2 - p^2}$$

जब $p < \omega$, तब $\tan\theta$ का मान काफी छोटा परन्तु धनात्मक होता है। इसका मान शून्य के काफी करीब होता है। अर्थात इस दशा में प्रणोदित दोलन व्यवहार में ड्राइविंग फोर्स के साथ कला संबद्ध होता है।

जब $p > \omega$, तब $\tan\theta$ काफी छोटा परन्तु ऋणात्मक होता है तथा इसका मान $\pi$ के काफी सन्निकट होता है। इस दशा में प्रणोदित दोलन ड्राइविंग बल के उल्टे कला संबद्ध होता है।

जब $p \approx \omega$, तब $\tan\theta = \infty$ अर्थात $\theta = \pi/2$ अर्थात अनुनाद की दशा में दोलन बल से हमेशा कोण पीछे रहता है। अर्थात बल अधिकतम होने पर विस्थापन न्यूनतम होता है। इसका उल्टा भी सत्य है।

**Q.11 : What is the difference between Amplitude Resonance and Velocity Resonance explain taking example of electrical oscillations. (06, 08,2013)**

**Sol$^n$.** When an alternating emf is applied to a circuit containing inductance capacitance and resistance then the current in the circuit oscillate with the frequency of the impressed emf when the frequency of the impressed emf is made equal to the natural frequency of the circuit, then the amplitude of the current becomes maximum. This phenomena is called **electrical resonance** and the process is known as **electrical oscillation.**

यदि किसी परिपथ में जिसमें प्रेरकत्व, धारिता तथा प्रतिरोध मौजूद हो उसमे एक

परिवर्तित emf आरोपित किया जाय तो परिपथ में बहने वाली विद्युत धारा आरोपित emf की आवृत्ति के बराबर हो जाती है तब धारा का आयाम महत्तम हो जाता है। यह धरना विद्युतीय अनुनाद तथा यह प्रक्रिया **विद्युतीय दोलन** कहलाती है।

The differential equation of force electrical oscillation is

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = E_0 \cos\omega t$$

or,

$$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}\frac{dQ}{dt} = \omega E_0 \cos\omega t \qquad \text{.............(1)}$$

Let the solution of equation (2) can be written as

$$I = I_0 \sin(\omega t - \phi)$$

Where $I_0$ and $\phi$ are constants.

$$\frac{dI}{dt} = \omega I_0 \cos(\omega t - \phi) \quad \text{and} \quad \frac{d^2 I}{dt^2} = -\omega^2 I_0 \sin(\omega t - \phi)$$

Substituting these values in equation (1), we get

$$-L\omega^2 I_0 \sin(\omega t - \phi) + R\omega I_0 \cos(\omega t - \phi) + \frac{I_0}{C}\sin(\omega t - \phi) = \omega E_0 \cos\omega t$$

$$\left(-L\omega^2 + \frac{1}{C}\right) I_0 \sin(\omega t - \phi) + R\omega I_0 \cos(\omega t - \phi) = \omega E_0 \cos\{(\omega t - \phi) + \phi\} \qquad \text{..... (2)}$$

Equation (2) is ture for all value of $t$. Hence equating the coefficients of $\sin(\omega t - \phi)$ and $\cos(\omega t - \phi)$, we get

$$\left(-L\omega^2 + \frac{1}{C}\right) I_0 = -\omega E_0 \sin\phi \qquad \text{..... (3)}$$

and

$$R\omega I_0 = \omega E_0 \cos\phi \qquad \text{...... (4)}$$

Squarring and adding equation (3) and (4), we get

$$I_0^2 \left[\left(-L\omega^2 + \frac{1}{C}\right) + R^2\omega^2\right] = \omega^2 E_0^2$$

or

$$I_0^2 \left[R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2\right] = E_0^2$$

or,

$$I_0 = \frac{E_0}{\left[R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2\right]} = \frac{E_0}{Z}$$

$I_0$ is the amplitude of the current and $\left[R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2\right]$ is the electrical impedance ($Z$) of the circuit.

**Amplitude Resonance :–** It is clear from the above equation that the impedance of the circuit is minimum when $\omega L$ is equal to $1/\omega C$. That is the amplitude off the current is maximum. It is the case of electrical resonance. Thus at resonance.

$$\omega L = \frac{1}{\omega C}$$

or, $$\omega = \frac{1}{\sqrt{LC}}$$

or, $$2\pi n = \frac{1}{\sqrt{LC}}$$

or, $$n = \frac{1}{2\pi\sqrt{LC}}$$

Therefore the amplitude of the current is maximum when the frequency of the applied voltage is equal to the natural frequency of the circuit with zero resistance. It is the condition of **Amplitude Resonance**.

**Velocity Resonance:–** The equation of current in the force electrical oscillation is

$$I = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}} \sin(\omega t - \phi)$$

Differentiality this equation, then we get–

$$\frac{dI}{dt} = \frac{E_0\omega}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}} \cos(\omega t - \phi)$$

$$\frac{dI}{dt} = \frac{E_0\omega}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}} \sin\{\omega t - (\phi - \pi/2)\}$$

It will be maximum when $\sin(\omega t - \phi + \pi/2) = 1$ The maximum valve is know as **velocity amplitude** $u_A$. Thus

$$u_A = \frac{E_0\omega}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}$$

It is the expression for the velocity amplitude of a driven harmonic oscillatior.

Thus, whatever be the damping, the velocity amplitude is maximum when the driving frequency is equal to the natural undemped frequency of the oscillation. This phenomenon is known as **Velocity Resonance**.

**Q.12 : Show that in the steady state of forced harmonic oscillation the average power supplied by the driving force is equal to that being dissipated by frictional force. (2012,2014)**

**Sol$^n$** : In order to maintain steady state oscillation of the system, the driving force must supply energy equal to that last in each cycle due to resistive force. Thus in steady state average power absorbed in a cycle is equal to the average power dissipated in a cycle.

The instantenous power $p$ absorbed by the oscillator is equal to the product of the instantenous driving force and the instantenous velocity. That is

किसी निकाय की स्थिर अवस्था में दोलन को बनाये रखने के लिए प्रेरित बल द्वारा प्रत्येक चक्र में प्रतिरोधी बल के कारण कम हुये ऊर्जा के बराबर ऊर्जा का स्थानान्तरण आवश्यक है। इस प्रकार स्थिर दोलन के प्रत्येक चक्र में औसत शक्ति अवशोषण उस चक्र में ऊर्जा क्षय के बराबर होता है।

दोलक द्वारा अवशोषित तात्कालिक शक्ति $P$ तात्कालिक प्रेरित बल तथा तात्कालिक वेग की गुणन के बराबर होता है।

$$P = (F_0 \sin pt)\left(\frac{dx}{dt}\right)$$

or,
$$P = \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} \sin pt[\cos(pt - \theta)]$$

or,
$$P = \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} \sin pt[\cos pt \cos\theta + \sin pt \sin\theta]$$

$$P = \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} [\sin pt \cos pt \cos\theta + \sin^2 pt \sin\theta]$$

The average input power in a cycle is given by -

$$P_{av} = \frac{1}{T}\int_0^T P(t)dt$$

$$P_{av} = \frac{1}{T}\int_0^T \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}} [\sin pt \cos pt \cos\theta + \sin^2 pt \sin\theta]\, dt \quad \text{...... (4)}$$

But we know that for one time perir :

$$\frac{1}{T}\int_0^T \sin pt \cos pt\, dt = 0 \quad \text{and} \quad \frac{1}{T}\int_0^T \sin^2 pt = \frac{1}{2}$$

Hence, the average power absorb is

$$P_{av} = \frac{mf_0^2 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}}\left(\frac{1}{2}\sin\theta\right) \quad \text{...... (5)}$$

As we know that - $\tan\theta = \dfrac{2rp}{(\omega^2 - p^2)}$

therefore, $\sin\theta = \dfrac{2rp}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}}$

Substiting the value of $\sin\theta$ in equation (5) then we get -

$$P_{av} = \frac{mf_0^2 p^2 r}{\sqrt{(\omega^2 - p^2)^2 + 4r^2p^2}}$$

This power is used in doing work against the damping force.

The rate of doing work (instantaneous power) $P'$ against the damping force is given by

$$P' = \text{Force} \times \text{Velocity}$$

or
$$P' = \left(2mr\frac{dy}{dx}\right).\frac{dy}{dx}$$

$$P' = 2mr\left(\frac{dy}{dx}\right)^2$$

or
$$P' = (2mr)\frac{f_0{}^2 p^2}{(\omega^2 - p^2) + 4r^2 p^2}\cos^2(pt - \theta)$$

The average of $\cos^2(pt - \theta)$ for one period T is

$$\frac{1}{T}\int_0^T \cos^2(pt - \theta)dt = \frac{1}{2}$$

Therefore, the average power dissipated is

$$P' = \frac{mrf_0{}^2 p^2}{(\omega^2 - p^2) + 4r^2 p^2} \quad \text{................(2)}$$

It is the same as the average power absorbed. Hence the power supplied by the driving force equals to the power dissipated.

**Q.13 : Calculate the quality factor and band width of a harmonic oscillator of mass 1 gm driven by a periodic force. (Given force constant $k = 10^4$ dyne/ cm and relaxation time $\tau = 1/2$ sec.**

**Sol$^n$ :** Given force constant $k = 10^4$ dyne/cm.

relaxation time $\tau = 1/2$ sec, mass of the oscillation = 1 gm

We know that $\omega = \sqrt{k/m}$

or
$$\omega = \sqrt{10^4/1} = 100 \text{ sec}^{-1}$$

Now the quality factor

$$Q = \omega\,\tau = 100 \times \frac{1}{2} = 50 \qquad \textbf{Ans.}$$

We also know that the qualification

$$Q = \frac{\omega}{\text{Band width}}$$

or
$$\text{Band width} = \frac{\omega}{Q} = \frac{100}{50} = 2 \qquad \textbf{Ans.}$$

**Q.14 : Discuss the phase behaviour of displacement and velocity of a forced mechanical oscillator. (2011,2014)**

**Sol$^n$ : Phase of the displacement and velocity of forced oscillation :-** When a system of mass $m$ having a natural angular frequecy $\omega$ oscillates under an external force $F_0 \sin pt$, the instanteneous displacement and velocity of the system are given by

$$x = \frac{f_0}{\sqrt{(\omega^2 - p^2)^2 + 4r^2 p^2}}\sin(pt - \theta) \quad \text{........ (1)}$$

Therefore, the instantaneous velocity

$$v = \frac{dx}{dt} = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2 p^2}}\cos(pt - \theta)$$

or, $$v = \frac{f_0 p}{\sqrt{(\omega^2 - p^2)^2 + 4r^2 p^2}} \sin\{pt - (\theta - \pi/2)\} \quad \text{........ (2)}$$

Where $f_0 = \frac{F_0}{m}$, $\omega^2 = \frac{p}{m}$ and $r = \frac{b}{2m}$ is a damping factor and

$$\tan\theta = \frac{2rp}{\omega^2 - p^2} \quad \text{....... (3)}$$

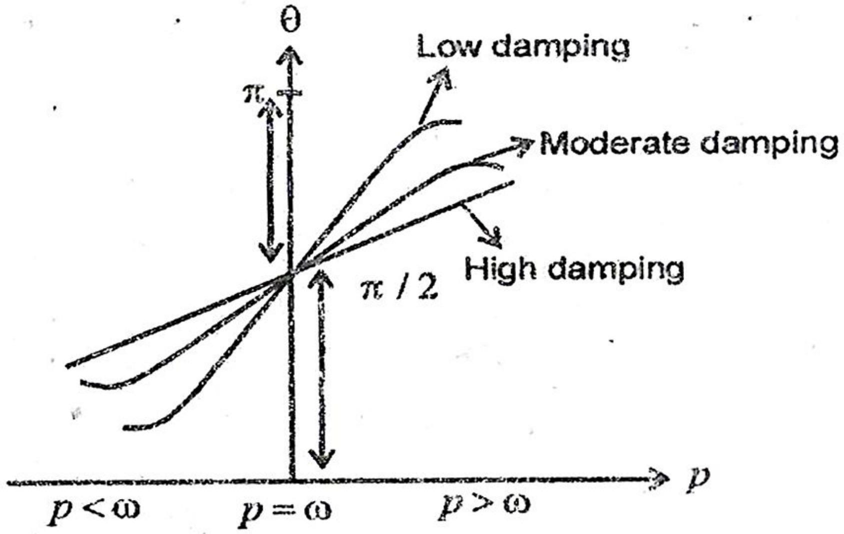
**This** shows that the phase difference depends on damping and also on the difference between $\omega$ and $p$. Now the following three cases may arise -

Case (i) : If $p < \omega$, $\tan\theta$ is +ve, $\theta$ lies between $0$ and $\pi/2$.

Case (ii) : If $p = \omega$, $\tan\theta$ is infinite, $\theta = \pi/2$.

Case (iii) : If $p > \omega$ $\tan\theta$ is negaive, $\theta$ lies between $\pi/2$ and $\pi$.

Thus the forced oscillations always lag behind the impressed force in phase by an angle between $0$ and $\pi$ depending on the value of $p$ relative to $\omega$. The variation of $\theta$ with $p$ are shown in the following figure.




If damping is very small ( $\pi \approx 0$ ), then

$$\tan\theta = \frac{2rp}{\omega^2 - p^2} = 0$$

It means that under this condition, the forced oscillations are practically in phase with the driving force.

When $p > \omega$, $\tan\theta$ is very small but now negative, so that $\theta$ i s very nearly to $\pi$, under this condition the forced oscillations are almost opposite in phase with the dividing force.

When $p = \omega$, $\tan\theta = \infty$, so that $\theta = \pi/2$. Thus at resolving driving force is maximum and vice versa.

Now according to equation (2) the velocity lags behind the driving force $F_0 \sin pt$ by an angle $\phi$ (says), where $\phi = \theta/2$.

When $p < \omega$, $\tan\theta$ is positive i.e. $\theta$ lies between 0 and $\pi/2$, so that $\phi$ is negative. This means that velocity leads the driving force.

When $p > \omega$, $\tan\theta$ is negative, i.e. $\theta$ lies between $\pi/2$ and $\pi$, so that $\phi$ is positive. This means that the velocity lags behind the driving force.

When $p = \omega$ (at velocity resonance), $\tan\theta = \infty$, so that -

$$\theta = \pi/2 \quad \text{and} \quad \phi = \pi/2 - \pi/2 = 0$$

Thus at resonance, the velocity is always in phase with driving force.nance, the ascillations always lag behind the force by $\pi/2$. It means that at resonance the displacement is minimum.

**Q 15 : In a LCR series circuit with L =0.2H and C=10.002 $\mu F$, Calculate the maximum value of resistance which makes it oscillatory. Determine the bandwidth at resonance using these value of L,C and R. (2014)**

Solⁿ: For an oscillatory circuit

$$\frac{R^2}{4L^2} < \frac{1}{LC}$$

or,
$$R^2 < \frac{4L}{C}$$

Give L = 0.2 H, $C = 10.002\mu F = 10.002 \times 10^{-6} F$

Therefore,
$$R^2 < \frac{4 \times 0.2}{10.002 \times 10^{-6}} = \frac{0.8}{10.002} \times 10^6$$

$$R^2 < 0.079 \times 10^6 \Rightarrow 7.9 \times 10^4$$

or,
$$R < 2.81 \times 10^2 \text{ ohm}$$

Thus, the resistance should be less than 281 ohm.

The bond width of resonance circuit is

$$B.W. = \frac{R}{L} = \frac{2.81 \times 10^2}{0.2} = 14.05 \times 10^2 = 1405$$ **Ans.**

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